Report no. 41

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Electron beam transport, betatron oscillation and phase space acceptance in ideal and present wiggler

1. Betatron Calculations

The betatron oscillation of the electron inside the wiggler was calculated by two ways: using the GPT simulations and by following analytical expressions:

The betatron wave number in the x and y directions:

$$k_{\beta x} = \sqrt{\frac{q_e \alpha_{rx}}{\gamma \beta_z c m_e}} \quad \text{and} \quad \lambda_{\beta_x} = \frac{2\pi}{k_{\beta_x}} \tag{1}$$

$$k_{\beta y} = \sqrt{\frac{k_\beta^2}{1-\left(\frac{k_{\beta x}}{k_w}\right)^2} - k_{\beta x}^2} \quad \text{and} \quad \lambda_{\beta_y} = \frac{2\pi}{k_{\beta_y}} \tag{2}$$

The results that we get with analytical expressions (1-2) are valid in paraxial approximation and weak long magnet gradient (the B_r of long magnets should be less than 5 kGauss) because of non-linear effects The results of betatron wave length calculations are summarized in tables 1 and 2. So the following calculations of the bettatron wave numbers and the wiggler phase-space acceptance was performed with GPT.

Table.1 The GPT calculations of the betatron wavelength for the ideal wiggler with long magnets with B_r=8 kGauss (α_{rx}=2.93). The analytical values of betatron wavelengths are: λx=290mm, λy=930mm

X-betatron amplitude [mm]	λx [mm]	Y-betatron amplitude [mm]	λy [mm]
1.7	0	0.1	930
2	290	0.25	924
3	290	0.5	910
4	290	1	880
5	280	2	740
6	270	3	600
7	265	4	500
8	264	5	420

Table.2 The GPT calculations of the betataron wavelength for the ideal wiggler with long magnets with B_r=4.45 kGauss (α_{rx}=1.61). The analytical values of betatron wavelengths are: λx=370mm, λy=400mm

X-betatron amplitude [mm]	λx	X-betatron amplitude [mm]	λy
2	400	0.5	370
3	400	1	370
4	400	2	370
5	400	3	350
6	400	4	325
7	400	5	300

2. The phase space acceptance of the present and ideal wiggler

In order to determine the phase-space acceptance of the present wiggle, we calculate the betatron wave numbers of single the electron, for different electron entrance conditions (entrance angle and amplitude). According to our waveguide dimensions the maximal electron trajectory amplitude should be: **-4mm<X<4mm; -6mm<Y<6mm**.

The calculation of the wiggler phase-space acceptance were performed for three following cases ideal wiggler with 23 and 17 equidistant placed long magnets with B_r=9.27kG and real wiggler with 17 equidistant placed magnet with average B_r=9.27kG. The results are summarized in Table 3, the calculation result are shown at figures 1-3.

Table 3 Relative phase-space acceptance for **-4mm<X<4mm; -6mm<Y<6mm**

Ideal wiggler 23 long mag.		Ideal wiggler 17 long mag.		Real wiggler 17 long mag.	
X' [mrad]	Y' [mrad]	X' [mrad]	Y' [mrad]	X' [mrad]	Y' [mrad]
90	35	80	50	70	45

a

b

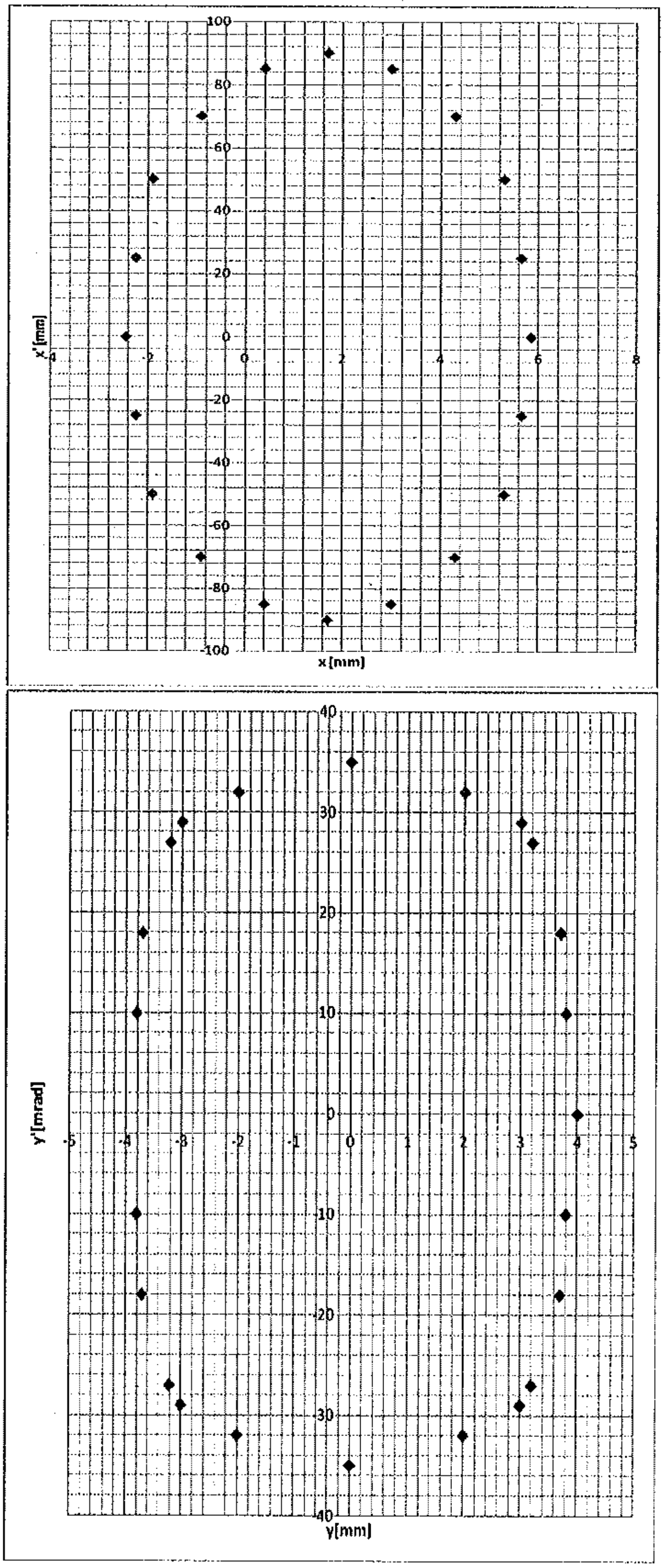


Fig. 1 a,b Angular acceptance of the ideal wiggler with 23 long magnets Br=9.27 kG

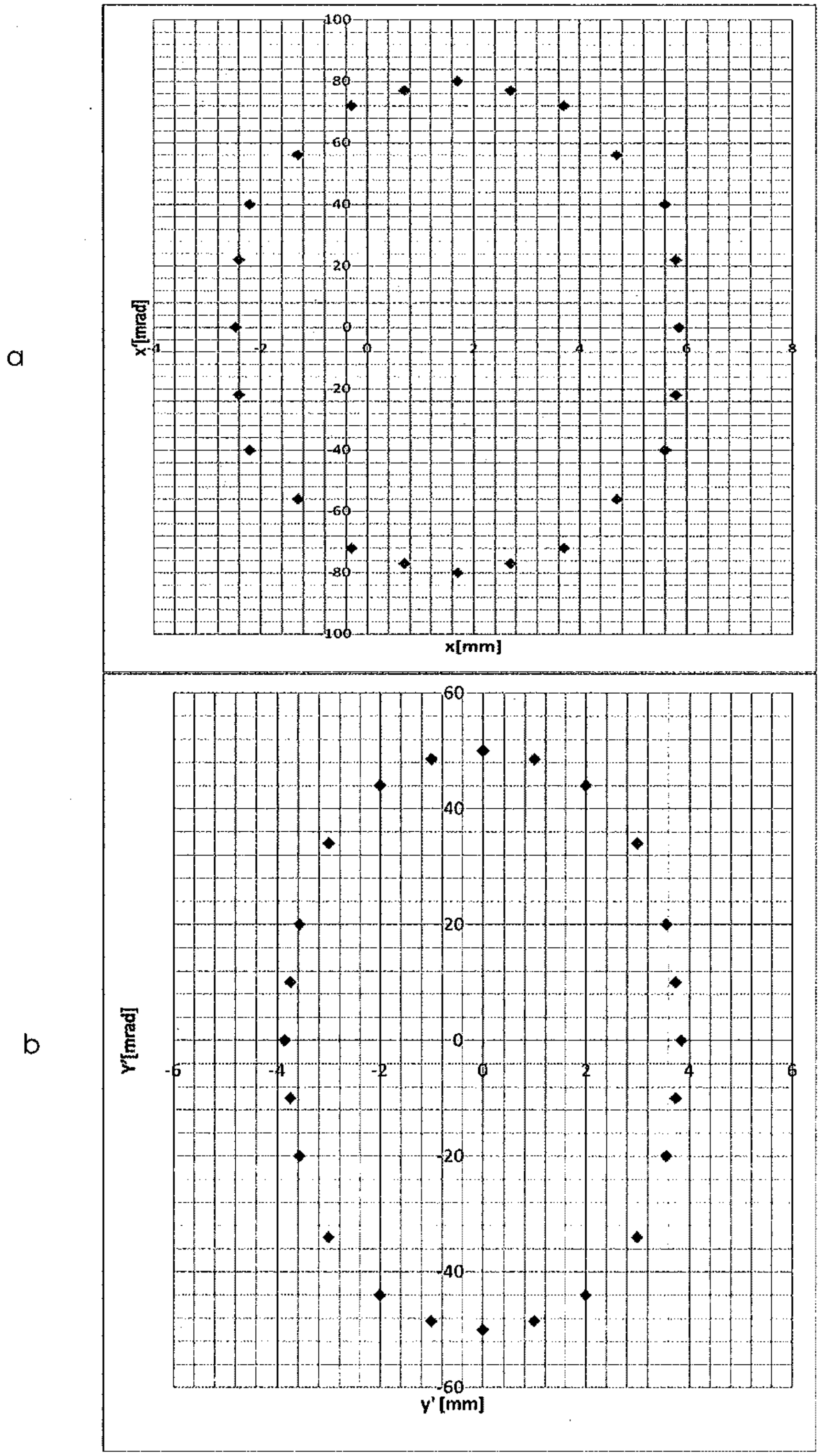


Fig. 2 a,b Angular acceptance of the ideal wiggler with 17 long magnets Br=9.27 kG a

b

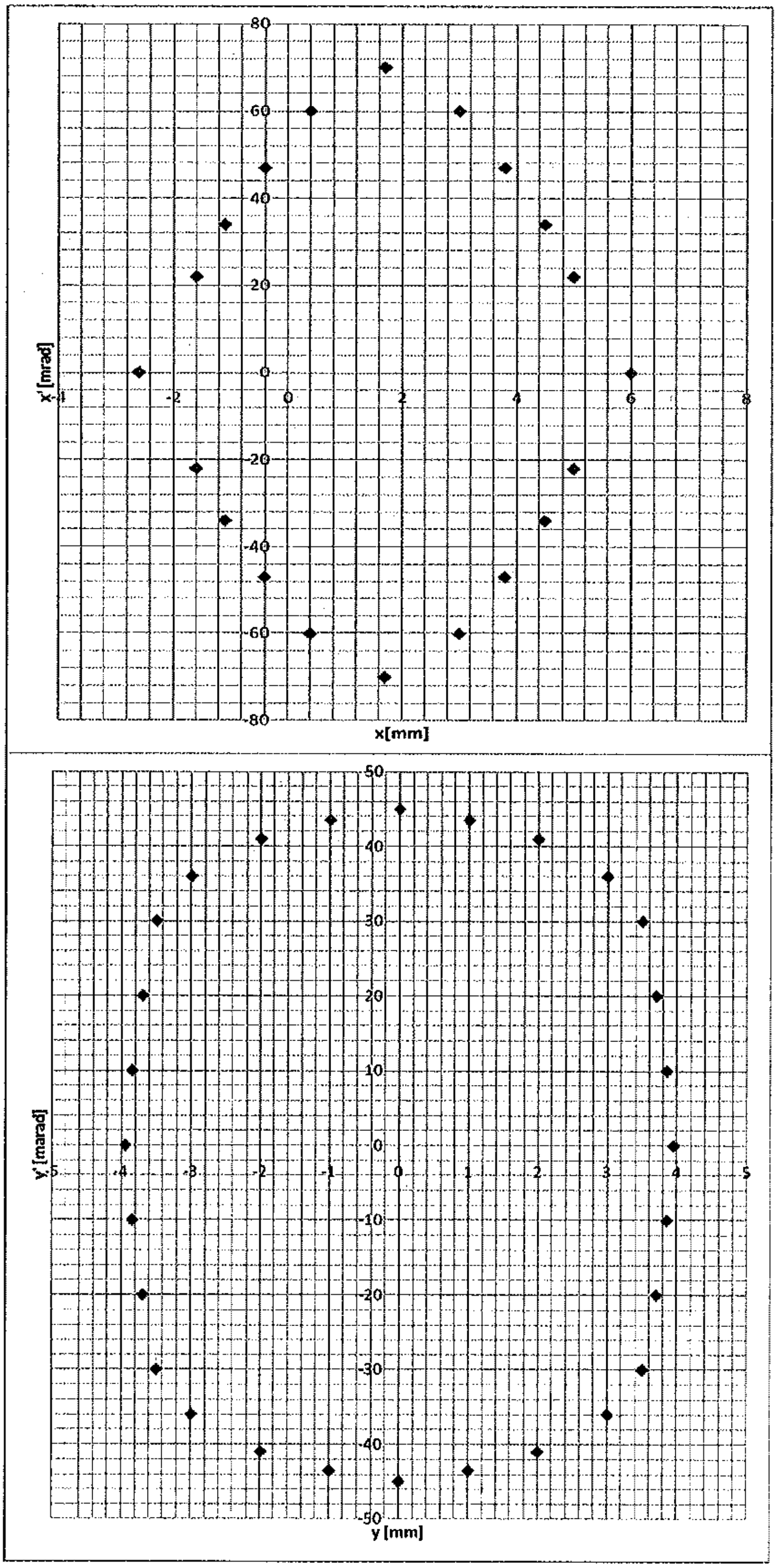
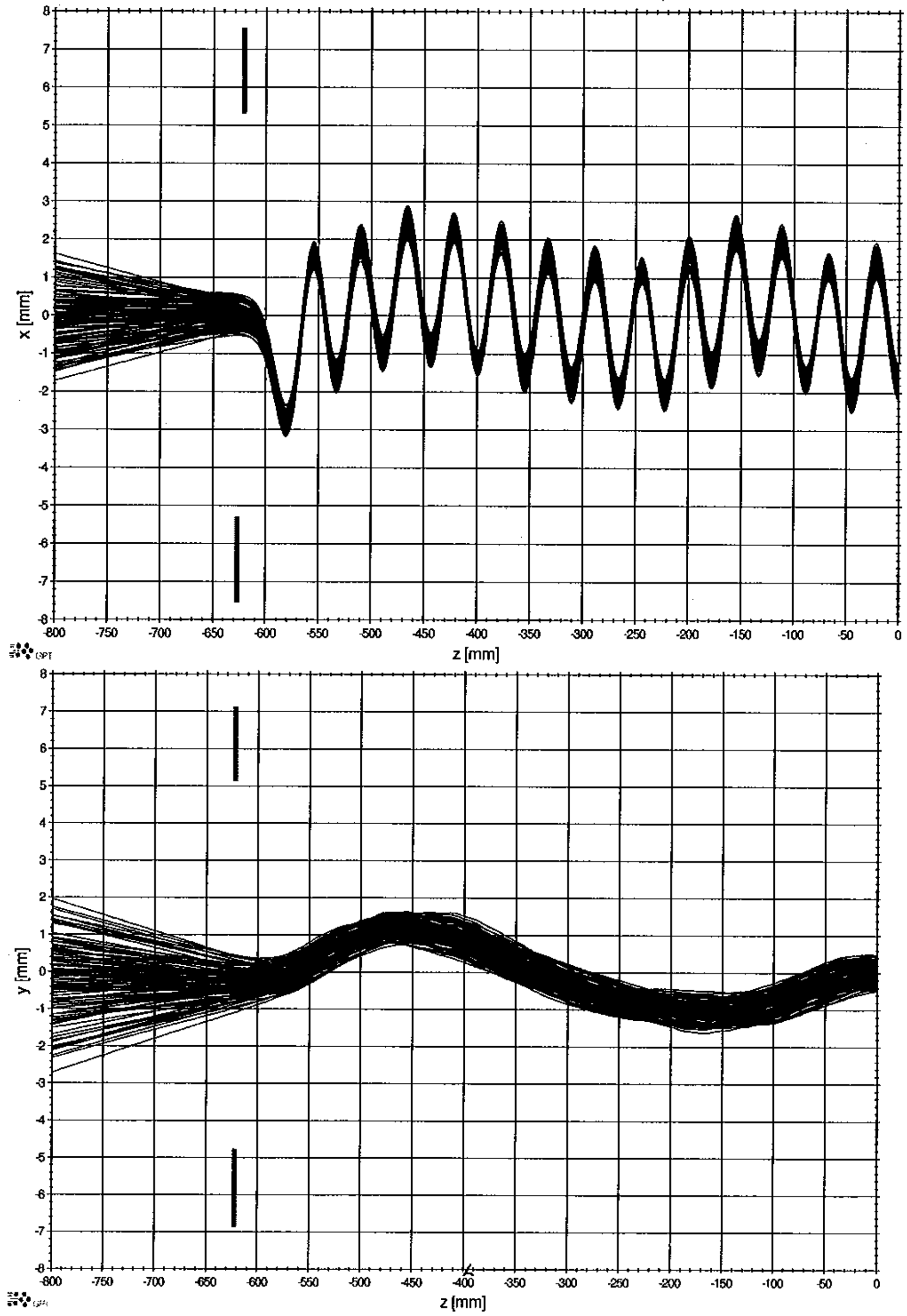


Fig. 3 a,b Angular acceptance of the real wiggler with 17 long magnets with average B_r=9,27kG

3. Electron beam transport in the present wiggler

The GPT simulation of the electron beam transport in the present wiggler is shown in the Figs. 4. The beam starts from x=-1.7mm, y=0mm, z= 0mm with X_b=0.5mm, Y_b=0.5mm and propagates forward and backward in the wiggler. We can see, that electron beam propagates almost without scalloping, and with weak betatron oscillations. The beam divergence angles are: $2\alpha_x$=~10mrad, $2\alpha_x$=~30mrad at the resonator apertures the electron beam has …. spatial diameters.



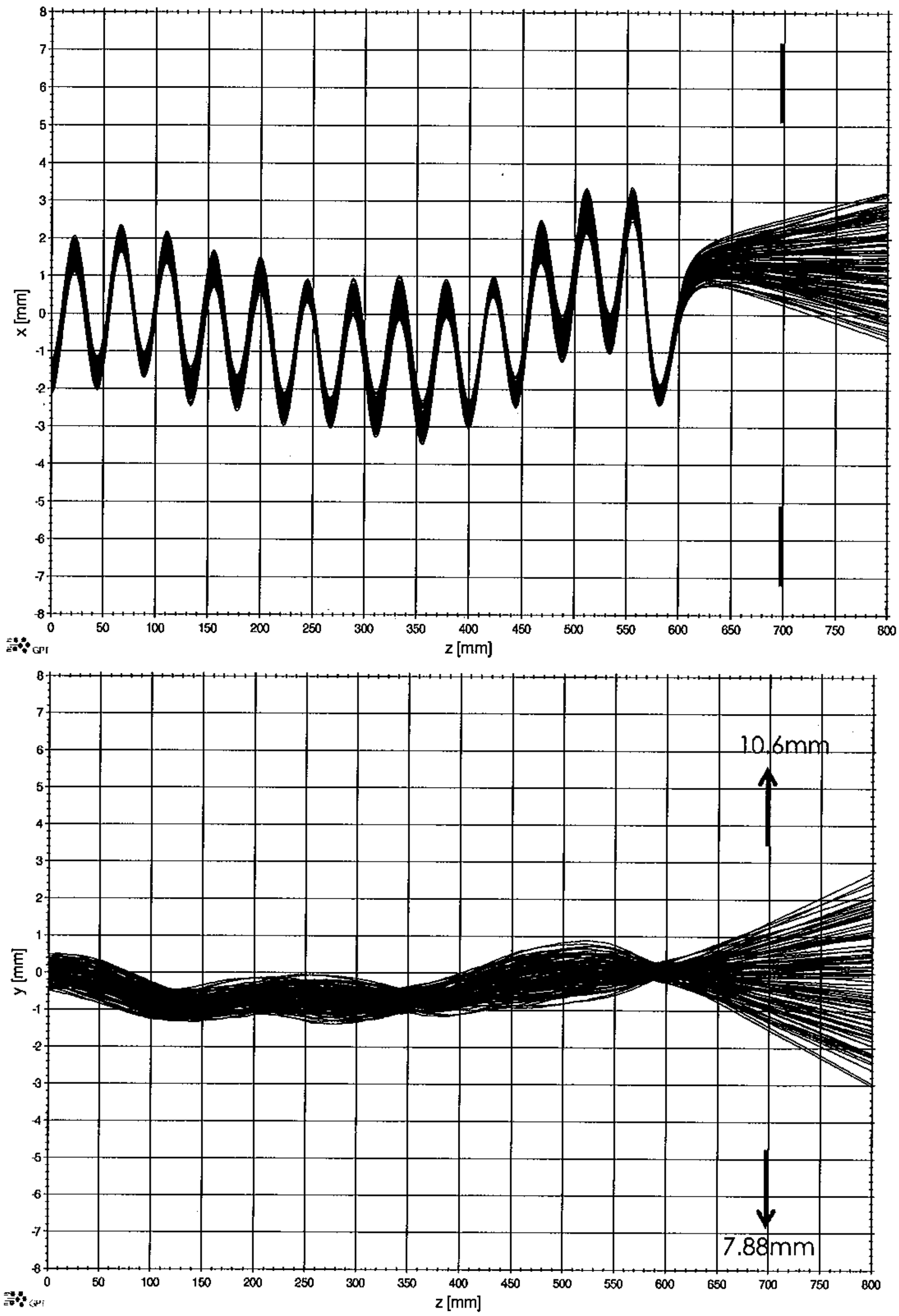


Fig 4. The GPT simulation of the electron beam transport in the present wiggler